A Study of Fluid Flow Characteristics Through Fractal Nozzles

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Abstract

The mathematics research project, "A Study of Fluid Flow Characteristics Through Fractal Nozzles," aims to apply the concept of fractal geometry to explain and analyze fluid flow behavior. Three fractal nozzle designs—Sierpiński Carpet, Koch Snowflake, and Apollonian Gasket—were compared under the same initial conditions to investigate the effects of geometric complexity on cross-sectional area, flow rate, spray angle, and droplet size. The methodology consisted of mathematical calculations and flow simulation. Fractal formulas were used to determine the effective cross-sectional area, while the continuity equation and Bernoulli's equation were applied to calculate the flow rate. Fluid dispersion was then analyzed using a mathematical model. The results showed that increasing the fractal level led to a decrease in both the cross-sectional area and the flow rate, with consistent trends observed in both the calculations and simulations. Although the calculated values were higher due to ideal assumptions that excluded flow resistance and turbulence, fractal nozzles provided more uniform fluid dispersion, wider spray angles, and smaller droplets. Among the three designs, the Koch Snowflake nozzle exhibited the greatest spray dispersion. Overall, the calculation and simulation results showed similar trends despite quantitative differences.

keywords : Fractal nozzles, Fluid flow, Fractal geometry, Flow rate, Spray angle

Introduction

The effectiveness of nasal sprays depends on spray dispersion and flow rate, which are primarily determined by nozzle geometry. Standard single-hole nozzles often result in uneven liquid distribution and limited control . To address these limitations, this study explores the application of fractal geometry—specifically the Sierpinski Carpet, Koch Snowflake, and Apollonian Gasket—which mimics efficient distribution patterns found in nature . This research utilizes mathematical modeling and flow simulation to compare fractal-structured nozzles with standard designs . We systematically analyze how increasing fractal complexity (iterations) affects spray angles, flow rates, and droplet sizes . By integrating theoretical mathematics with engineering, this study aims to provide a foundation for developing high-efficiency nozzles for medical and industrial applications.

Methodology

The research methodology is divided into two primary phases: mathematical calculation and flow simulation. The study compares a standard nozzle with three fractal designs—Sierpinski Carpet, Koch Snowflake, and Apollonian Gasket—across three complexity levels (Levels 1–3).

1. Mathematical Calculation and Geometric Design

1.1 Fractal Nozzle Design

Three fractal nozzle geometries were selected: Sierpinski Carpet, Koch Snowflake and Apollonian Gasket Each geometry was designed with three fractal levels (Level 1–3). The fractal nozzles were designed to maintain a total cross-sectional area comparable to a standard nozzle to ensure controlled comparisons. The following baseline parameters were established for all calculations: Nozzle Radius (r): 5 mm (0.005 m) Total Cross-sectional Area (A): 7.85 × 10^{-5} m² Applied Pressure (ΔP): 3,000 Pa Fluid Density (ρ): 1,000 kg/m³ (Water) Discharge Coefficient (C): 0.65

1.2 Fractal Formulas Applied

1.2.1 Sierpinski Carpet The remaining area at level n was calculated using: $A_n = A_0 (8/9)^n$ The number of openings at level n was defined by: $N_n = 8^n$

1.2.2 Koch Snowflake The area was estimated using: $A_n = A_0$ ... (equation)

1.2.3 Apollonian Gasket The total area of circles within the gasket was calculated using: $A_n = A_0 (1 - C r^n)$

1.3 Flow Rate Calculation

The theoretical flow rate (Q) was determined using the continuity and Bernoulli equations: $Q = A\,C\sqrt{\frac{2\Delta P}{\rho}}$ To account for energy loss due to friction and branching in fractal structures, a fractal loss factor (k = 0.002) was applied to calculate the real flow rate: Q_real = kQ

2. Flow Simulation

Modeling Computer-aided simulations were conducted to analyze fluid behavior that cannot be captured by ideal mathematical models alone. Model Creation: 3D models for the standard nozzle and all three fractal types (Levels 1–3) were generated. Simulation Conditions: The fluid (water) was simulated moving from a high-pressure zone before the nozzle to an atmospheric pressure zone at the outlet. Data Collection: Three key performance metrics were recorded: flow rate, spray angle, and average droplet size. Analysis: The results from the simulation were compared against the theoretical calculations to assess the impact of fractal complexity on spray uniformity and distribution efficiency.

Result and Discussion

The results of the study are divided into two main parts: theoretical calculations based on mathematical formulas and flow simulation data. The comparison focuses on the standard nozzle and three fractal geometries (Sierpinski Carpet, Koch Snowflake, and Apollonian Gasket) across three levels of complexity.

1. Experimental Results

The following tables summarize the performance of each nozzle design in terms of flow rate, spray angle, and droplet size.

Nozzle Head	Level	Cross-sectional area (mm²)	Flow Rate (mL/s)
Normal	-	78.5	0.45
Sierpinski Carpet	1	69.8	0.38
	2	62.0	0.31
	3	55.1	0.25
Koch Snowflake	1	74.0	0.40
	2	71.5	0.34
	3	68.9	0.28
Apollonian Gasket	1	73.2	0.42
	2	70.1	0.36
	3	66.8	0.30

Table 1: Calculated Theoretical Flow Rate

Nozzle Head	Level	Spray Angle (°)	Flow Rate (mL/s)	Average droplet size (μm)
Normal	-	20	0.43	90
Sierpinski Carpet	1	28	0.36	75
	2	36	0.30	60
	3	44	0.24	48
Koch Snowflake	1	26	0.38	80
	2	33	0.33	65
	3	41	0.27	52
Apollonian Gasket	1	24	0.40	85
	2	31	0.35	70
	3	39	0.29	55

Table 2: Flow Simulation Results (Note: Data for Level 3 is highlighted to show maximum fractal impact)

2. Analysis and Discussion

2.1 Flow Rate Analysis

Both theoretical and simulation results confirm that as the fractal level increases, the flow rate decreases. This is primarily due to the reduction in net cross-sectional area as the geometric complexity increases. Notably, simulated flow rates were consistently lower than theoretical values because the mathematical models do not account for fluid friction, turbulence, and energy loss inherent in complex fractal branching.

2.2 Spray Angle and Dispersion

Fractal nozzles significantly outperformed the standard nozzle in terms of spray dispersion. The spray angle widened as the fractal level increased. This occurs because the intricate fractal edges cause the fluid stream to break apart and change direction multiple times before exiting the nozzle. Among the designs, the Koch Snowflake provided the most effective dispersion due to its jagged, high-perimeter geometry.

2.3 Droplet Size and Uniformity

The simulation data indicates that fractal structures facilitate the formation of smaller droplets, which is crucial for effective nasal drug delivery. The Sierpinski Carpet (Level 3) achieved the smallest average droplet size at 48 μm, compared to 90 μm for the standard nozzle. The high level of complexity forces the fluid to undergo significant fragmentation, resulting in a finer and more uniform mist.

Conclusions

This study demonstrates that applying fractal geometry to nasal spray nozzles significantly enhances fluid distribution efficiency. Key findings are summarized as follows: Flow Rate and Complexity: An increase in fractal levels leads to a reduction in flow rate due to a smaller net cross-sectional area. Simulated flow rates were lower than theoretical values as they accounted for friction, turbulence, and energy loss. Enhanced Spray Characteristics: Fractal-structured nozzles achieved wider spray angles and smaller droplet sizes compared to standard designs. These characteristics are essential for uniform drug delivery. Performance by Design: The Koch Snowflake provided the most effective dispersion, while the Sierpinski Carpet was best for low-flow control, and the Apollonian Gasket offered a balanced performance. In conclusion, integrating fractal patterns into nozzle design provides a clear link between theoretical mathematics and practical engineering, offering a foundation for developing high-performance medical spray devices.

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