



A Trigonometric and Analytical Geometry Approach to Mitigating Moiré Patterns in Digital Display Photography

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Abstract

This study aims to investigate and develop a mathematical model of the moiré Effect using principles of analytic geometry and trigonometry. The objective is to determine suitable camera rotation angles and shooting conditions for reducing unwanted Moiré fringe patterns in digital display photography. The methodology consists of mathematical modeling and computer simulation. The pixel grid of a digital display is represented by the cosine function $f(x) = \cos\left(\frac{2\pi}{d_1}x\right)$, where d_1 denotes the pixel pitch of the display. Meanwhile, the camera sensor array with a pixel pitch of d_2 is rotated by an angle θ and modeled using axis rotation matrices as $g(x,y) = \cos\left(\frac{2\pi}{d_2}x \cos \theta + y \sin \theta\right)$.

The interaction between the display and camera sensor grids produces the moiré pattern, which can be expressed as $M(x,y) = f(x) \cdot g(x,y)$. By applying product-to-sum trigonometric identities, the resulting pattern can be separated into distinct frequency components as follows:

$$M(x,y) = \frac{1}{2} \left[\cos \left(2\pi \left(\left(\frac{1}{d_1} - \frac{\cos \theta}{d_2} \right) x - \frac{\sin \theta}{d_2} y \right) \right) + \cos \left(2\pi \left(\left(\frac{1}{d_1} + \frac{\cos \theta}{d_2} \right) x + \frac{\sin \theta}{d_2} y \right) \right) \right]$$

From this mathematical analysis, the difference-frequency component, $f_{\text{moire}} = \left(\frac{1}{d_1} - \frac{\cos \theta}{d_2}, -\frac{\sin \theta}{d_2} \right)$, is identified as the main factor responsible for the large-scale Moiré fringes visible to the human eye. Simulations were performed using GeoGebra to observe changes in fringe patterns under different camera rotation angles. The results show that increasing the rotation angle decreases $\cos \theta$, causing an increase in difference-frequency magnitude and reducing the size and visibility of Moiré fringes. The experimental observations agree with theoretical predictions, indicating that a camera rotation angle within the range of $15^\circ - 30^\circ$ can significantly reduce the Moiré Effect. This study demonstrates the effectiveness of mathematical modeling for analyzing and minimizing Moiré artifacts in digital imaging systems.

Keywords: Moiré Effect, Analytic Geometry, Rotation of Axes, Trigonometry, Difference Frequency Component