A Trigonometric and Analytical Geometry Approach to Mitigating Moiré Patterns in Digital Display Photography

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Abstract

This study aims to investigate and develop a mathematical model of the moiré Effect using principles of analytic geometry and trigonometry. The objective is to determine suitable camera rotation angles and shooting conditions for reducing unwanted Moiré fringe patterns in digital display photography. The methodology consists of mathematical modeling and computer simulation. The pixel grid of a digital display is represented by the cosine function $f(x) = \cos\left(\frac{2\pi}{d_1}x\right)$, where d_1 denotes the pixel pitch of the display. Meanwhile, the camera sensor array with a pixel pitch of d_2 is rotated by an angle θ and modeled using axis rotation matrices as $g(x,y) = \cos\left(\frac{2\pi}{d_2}\ x\cos\theta + y\sin\theta\ \right)$.

The interaction between the display and camera sensor grids produces the moiré pattern, which can be expressed as $M(x,y) = f(x)\cdot g(x,y)$. By applying product-to-sum trigonometric identities, the resulting pattern can be separated into distinct frequency components as follows:

$$M(x,y) = \frac{1}{2}\left[\cos\left(2\pi\left(\left(\frac{1}{d_1} - \frac{\cos\theta}{d_2}\right)x - \frac{\sin\theta}{d_2}y\right)\right) + \cos\left(2\pi\left(\left(\frac{1}{d_1} + \frac{\cos\theta}{d_2}\right)x + \frac{\sin\theta}{d_2}y\right)\right)\right]$$

From this mathematical analysis, the difference-frequency component, $f_{moire} = \left(\frac{1}{d_1} - \frac{\cos\theta}{d_2}, -\frac{\sin\theta}{d_2}\right)$, is identified as the main factor responsible for the large-scale Moiré fringes visible to the human eye. Simulations were performed using GeoGebra to observe changes in fringe patterns under different camera rotation angles. The results show that increasing the rotation angle decreases $\cos\theta$, causing an increase in difference-frequency magnitude and reducing the size and visibility of Moiré fringes. The experimental observations agree with theoretical predictions, indicating that a camera rotation angle within the range of $15^\circ - 30^\circ$ can significantly reduce the Moiré Effect. This study demonstrates the effectiveness of mathematical modeling for analyzing and minimizing Moiré artifacts in digital imaging systems.

Keywords: *Moiré Effect, Analytic Geometry, Rotation of Axes, Trigonometry, Difference Frequency Component*

Introduction

In the modern digital era, electronic display devices such as computer monitors, televisions, smartphones, and LED screens have become essential tools for communication and information sharing. However, when these displays are photographed using digital cameras, unwanted colored ripple patterns or grid-like distortions may appear, reducing image quality and accuracy. This phenomenon is known as the Moiré Effect. From a physics and optical perspective, the Moiré Effect is caused by the interaction between two periodic structures with similar spatial frequencies: the display pixel grid and the camera sensor array, commonly represented by the Color Filter Array (CFA) or Bayer Pattern. When these structures overlap with slight differences in orientation or frequency, incorrect sampling or aliasing occurs, producing low-frequency interference patterns visible as Moiré fringes.

Although existing solutions, such as image post-processing and Optical Low-Pass Filters (OLPFs), can reduce this problem, they may increase costs or decrease image sharpness. Therefore, preventing Moiré formation through proper camera positioning provides a practical and efficient alternative.

Mathematically, the pixel grid and camera sensor array can be modeled as periodic geometric structures using analytic geometry and trigonometric functions. Rotation matrices, coordinate transformations, and trigonometric identities allow the relationship between these structures and the resulting Moiré patterns to be analyzed.

This project aims to develop a mathematical model of the Moiré Effect using analytic geometry and trigonometry. The model will be simulated using GeoGebra to determine suitable camera rotation angles and shooting conditions that minimize Moiré patterns. The findings demonstrate the application of pure mathematics in explaining real-world optical phenomena and may provide practical guidelines for improving digital photography systems.

Objectives

1. To develop a mathematical model of the Moiré Effect using analytic geometry and trigonometry.
2. To determine suitable camera rotation angles and shooting conditions for reducing Moiré patterns in digital display photography.

Scope of the Project

This study focuses on LED and LCD displays with regular pixel structures. A two-dimensional Cartesian coordinate system and trigonometric functions are applied to model the interaction between pixel grids and camera sensor arrays.

Research Hypothesis

Adjusting the camera to an appropriate mathematically determined rotation angle can significantly reduce the visibility of Moiré patterns in digital display photographs.

Expected Benefits

This project provides mathematical models and practical guidelines for selecting suitable camera angles and shooting conditions. It also enhances understanding of how analytic geometry and trigonometry can be applied to solve technological problems in real-world situations.

Methodology

The research methodology was divided into two main stages: (1) Mathematical Modeling, (2) Computer Simulation using GeoGebra. Each stage was designed to analyze the formation of Moiré patterns and determine suitable camera conditions for reducing their visibility.

1. Mathematical Modeling

This stage converts the physical structures of the digital display and camera sensor into mathematical representations to analyze Moiré interference patterns.

1.1 Modeling the Digital Display (Screen Grid)

The digital display is represented in a two-dimensional Cartesian coordinate system (x, y). The vertical pixel structure is assumed to have uniform spacing with a pixel period d_1. The intensity distribution of the display grid is modeled using a periodic cosine function:

$$f(x) = \cos\left(\frac{2\pi}{d_1} x\right)$$

1.2 Modeling the Rotated Camera Sensor (Sensor Grid)

When the camera is rotated by an angle θ, the camera coordinate system undergoes a geometric transformation relative to the screen coordinate system. Using the rotation matrix concept in analytic geometry, the sensor grid with pixel spacing d_2 is represented as:

$$g(x, y) = \cos\left(\frac{2\pi}{d_2}\ x\cos\theta + y\sin\theta\right)$$

1.3 Trigonometric Analysis of moiré Superposition

The interaction between the display grid and the camera sensor grid is described by the product of the two periodic functions:

$$M(x, y) = f(x) \cdot g(x, y)$$

Applying the product-to-sum trigonometric identity,

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

, yields:

$$M(x, y) = \frac{1}{2}\left[\begin{aligned}&\cos\left(2\pi\left(\left(\frac{1}{d_1} - \frac{\cos\theta}{d_2}\right)x - \frac{\sin\theta}{d_2}y\right)\right)\\ &+\cos\left(2\pi\left(\left(\frac{1}{d_1} + \frac{\cos\theta}{d_2}\right)x + \frac{\sin\theta}{d_2}y\right)\right)\end{aligned}\right]$$

difference-frequency component

$$\left(\frac{1}{d_1} - \frac{\cos\theta}{d_2}, -\frac{\sin\theta}{d_2}\right)$$

is responsible for generating the low-frequency interference patterns that appear as large-scale

Moiré fringes. This relationship is used to determine the camera rotation angle that increases the difference-frequency magnitude and reduces the visibility of Moiré patterns

2. Computer Simulation

The mathematical model was simulated using GeoGebra to visualize Moiré pattern formation under different camera rotation angles.

2.1 Parameter Setup

A slider α, representing the camera rotation angle, was created with a range from 0° to 90° and an increment of 0.5°. Additional parameters representing pixel spacing were defined as: $d_1 = 0.20, d_2 = 0.22$

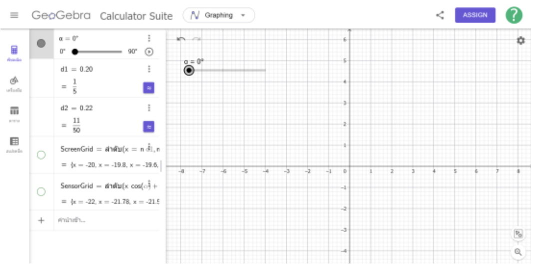

Fig. 1 Parameter Setup

2.2 Constructing the Static Screen Grid

The digital display pixel grid was generated using the GeoGebra Sequence command

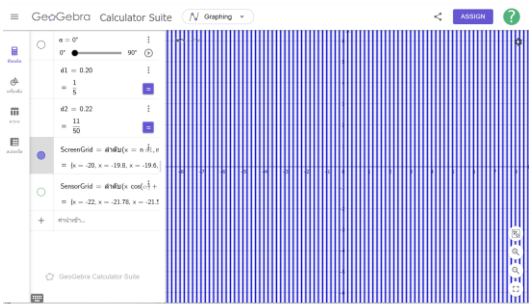

Fig. 2 Static Screen Grid

2.3 Constructing the Rotated Sensor Grid

The camera sensor grid was constructed using the rotation equation:

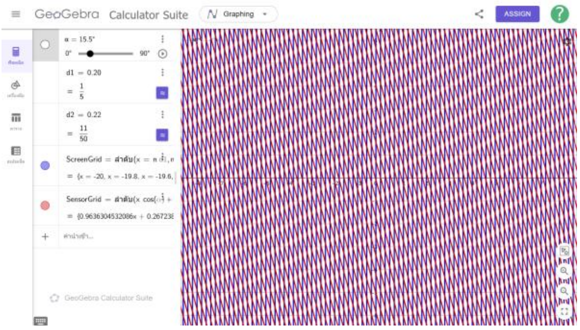

Fig. 3 Moiré Superposition

The screen grid was displayed as blue lines, while the rotated sensor grid was displayed as red lines, allowing the interaction between the two periodic structures to be observed.

2.4 Observation and Data Collection

The rotation angle α was gradually increased from 0° to 90°. The resulting intersection patterns and bright–dark Moiré fringes were observed and recorded. Representative images were captured at rotation angles of 0°, 15°, 30°, and 45° for further analysis.

Result and Discussion

1. Mathematical Analysis Results

Based on the principles of analytic geometry, particularly coordinate rotation, and the trigonometric product-to-sum identity, described in the Mathematical Modeling section (Section 1). the interference relationship between the digital display pixel grid and the camera sensor grid was mathematically analyzed. The resulting Moiré superposition function is expressed as:

$$M(x,y)=\frac{1}{2}\left[\begin{array}{l}\cos\left(2\pi\left(\left(\frac{1}{d_1}-\frac{\cos\theta}{d_2}\right)x-\frac{\sin\theta}{d_2}y\right)\right)\\+\cos\left(2\pi\left(\left(\frac{1}{d_1}+\frac{\cos\theta}{d_2}\right)x+\frac{\sin\theta}{d_2}y\right)\right)\end{array}\right]$$

Where d_1 represents the pixel spacing of the digital display, d_2 represents the pixel spacing of the camera sensor, and θ represents the rotation angle between the two periodic structures.

The difference-frequency component, which is responsible for generating the large-scale moiré fringes visible to the human eye, is represented as:

$$f_{moire}=\left(\frac{1}{d_1}-\frac{\cos\theta}{d_2},-\frac{\sin\theta}{d_2}\right)$$

When the camera is aligned parallel to the display surface $(\theta=0^\circ)$, the trigonometric values become $\cos(0^\circ)=1$ and $\sin(0^\circ)=0$ Therefore, the frequency component exists only along the x-axis. When the pixel spacing of the display and camera sensor are approximately equal $(d_1 \sim d_2)$, the difference frequency approaches zero, resulting in a large spatial wavelength.

As the rotation angle increases $0^\circ<\theta<90^\circ$, the value of $\cos\theta$ decreases, causing the term:

$$\frac{1}{d_1}-\frac{\cos\theta}{d_2}$$

to increase. Additionally, the y-direction frequency component:

$$-\frac{\sin\theta}{d_2}$$

also increases in magnitude. These changes cause the Moiré fringes to rotate, become distorted, and decrease in spatial size. When the spatial frequency exceeds the visual resolution limit of the human eye, the Moiré pattern becomes significantly less visible.

2. Model Analysis and Comparison with Real-World Applications

The mathematical model shows that camera rotation directly influences the interference frequency between the display pixel grid and the camera sensor array. When the camera is parallel to the display, the frequency difference between the two structures is minimal, resulting in large-scale Moiré fringes. Increasing the rotation angle increases the difference-frequency magnitude due to the reduction of $\cos\theta$. Consequently, the spatial wavelength of the Moiré fringes decreases, causing the patterns to become smaller and less noticeable. According to the model, a camera rotation angle within approximately $15^\circ-30^\circ$ is an appropriate range for reducing the visibility of the Moiré Effect while maintaining image quality. This result agrees with practical photography conditions, where slightly tilting a camera when capturing digital screens can reduce unwanted Moiré patterns.

However, the optimal angle depends on several factors, including pixel density, sensor structure, and display technology. Therefore, the obtained angle range should be considered an estimation under the defined mathematical conditions rather than a universal value.

Conclusions

1. Conclusion of Findings

This project aimed to develop a mathematical model of the Moiré Effect using analytical geometry and trigonometry.

The mathematical model and GeoGebra simulation showed that increasing the camera rotation angle reduces the visibility of Moiré patterns by increasing the difference frequency between the screen pixel grid and the camera sensor grid. Under the conditions of this study, a rotation angle of approximately 15°–30° was found to be a suitable range for minimizing Moiré patterns. However, this value is an estimation and may vary depending on the display and camera characteristics.

The study demonstrates that analytical geometry and trigonometric principles can be effectively applied to explain and solve practical problems in digital imaging technology.

2. Limitations

This study has several limitations. Different display technologies have varying pixel densities, which may affect the optimal rotation angle for reducing Moiré patterns. Additionally, differences in camera sensor structures and image-processing systems may influence the observed patterns.

The proposed model is based on a two-dimensional representation and does not include factors such as lens perspective, depth, and three-dimensional imaging effects.

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