



**FIBONACCI SPIRAL IN THE CARTESIAN COORDINATE SYSTEM:
IMAGINATION FROM GRAPH TO INFINITE LENGTH AND GOLDEN RATIO**

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Abstract

This project originated from an interest in the Fibonacci sequence (1,1,2,3,5, 8,...), a simple sequence possessing fascinating mathematical characteristics and appearing widely in nature, such as in the patterns of flower petals, tropical storms, and galaxies. The authors began by imagining the mapping of Fibonacci numbers as points on the axes of a Cartesian coordinate system, plotted in an infinitely spiraling sequence. By continuously connecting each pair of points with line segments, a geometric shape resembling a spiral was formed, referred to in this project as the “Fibonacci Spiral on the Cartesian Plane.” From this figure, the authors observed patterns and formulated conjectures, leading to the proof of four interesting theorems: 1) the length of the n -th line segment in the spiral can be determined from the square root of a specific Fibonacci number; 2) the ratio of the lengths of adjacent line segments converges to the Golden Ratio (ϕ); 3) the ratio of the areas of adjacent triangles within the spiral converges to the square of the Golden Ratio; and 4) adjacent line segments are perpendicular to each other, which implies that segments separated by two orders are parallel. This project serves as a clear example that mathematics is not merely a subject in textbooks but the language nature uses to design itself. From ordinary straight lines connecting points, the project reveals the profundity of infinite geometry through observation, questioning, and the search for hidden meaning in simple things. Beyond theoretical understanding, the developed model could support future applications, such as modeling the eyes of storms or galaxies, while serving as a comprehensive learning platform that encompasses observation, analysis, proof, creative thinking, and the use of technology to visualize results.

Keywords: Fibonacci Numbers, Golden Ratio, Fibonacci Spiral, Infinite Length